Legibility and Predictability of Robot Motion

Central Insight :

Predictability and legibility are fundamentally different and often contradictory properties of motion

I. Formalizing legibility and predictability

Definition 1 : Legible motion is motion that enables an observer to quickly and confidently infer the correct goal G.

Definition 2 : Predictable motion is motion that matches what an observer would expect, given goal G.

all trajectories

Legibility : Observer makes inference $I_L : \Xi \to G$

→ legible motion : confidently infer the correct goal G given a snippet of trajectory, $\xi_{S\to Q}$, $Q = \xi(t)$

$$I_L(\xi_{S\to Q}) = G$$

Predictability : Observer knows the goal, expect certain action from the robot, $I_P : G \to \Xi$

→ Predictable motion : $I_P(G) = \xi_{S\to G}$

II. Modeling Predictable Motion

- I_P : robot acts "rationally", "efficiency" & cost function.

$$C : \Xi \to \mathbb{R}^+$$

observer's expectation

lower costs → more "efficient" trajectories : $I_P(G) = \arg\min_{\xi \in \Xi_{S\to G}} C(\xi)$

- Predictability score $\in [0, 1]$

predictability$(\xi) = \exp(-C(\xi))$ (goal : maximize this score)

III Modeling Legible Motion.

- I_L : which end state will be most "efficiently" achieved by $\xi_{S\to Q}$

→ $I_L(\xi_{S\to Q}) = \arg\max_{G \in \mathcal{G}} P(G | \xi_{S\to Q})$

→ can be uniform

Using Baye's Rule : $P(G|\xi_{S\to Q}) \propto P(\xi_{S\to Q}|G)P(G)$

finding trajectory given G

$$P(\xi_{S\to Q}|G) = \frac{\int_{\xi_{Q\to G}} P(\xi_{S\to Q\to G})}{\int_{\xi_{S\to G}} P(\xi_{S\to G})}$$ → all trajectories from $S \to Q \to G$ / → all trajectories from $S \to G$

$$P(\xi_{S\to Q}|G) = \frac{P(\xi_{S\to Q}) \int_{\xi_{Q\to G}} P(\xi_{Q\to G})}{\int_{\xi_{S\to G}} P(\xi_{S\to G})}$$ assuming $P(\xi_{X\to Y\to Z}) = P(\xi_{X\to Y})P(\xi_{Y\to Z})$ separable

Principle of maximum entropy:

$$P(\xi_{S\to Q}|G) \propto \frac{\exp(-C(\xi_{S\to Q}))\int_{\xi_{Q\to G}}\exp(-C(\xi_{Q\to G}))}{\int_{\xi_{S\to G}}\exp(-C(\xi_{S\to G}))}$$

Using Laplace's method to approximate probabilities. If C is quadratic, its Hessian is constant

$$\to \int_{\xi_{X\to Y}}\exp(-C(\xi_{X\to Y})) \approx k\exp(-C(\xi^*_{X\to Y}))$$ optimal trajectory

$$\to P(G|\xi_{S\to Q}) \propto \frac{\exp(-C(\xi_{S\to Q}) - C(\xi^*_{Q\to G}))}{\exp(-C(\xi^*_{S\to G}))} P(G)$$

- $\text{legibility}(\xi) = \dfrac{\int P(G^*|\xi_{S\to\xi(t)}) f(t)\,dt}{\int f(t)\,dt}$ $f(t)$ = trajectory weight $= T - t$ (T = duration of the trajectory)
- To prevent the robot from going too far, add a regularizer.

$$L(\xi) = \text{legibility}(\xi) - \lambda C(\xi)$$

Implication:

Collaborative robots must be legible in all collaboration paradigms